sions, or differing considerably in longitude, ψ_W being the latitude of the western and ψ_E of the eastern station. Let also h_W and h_E be the corresponding hour-angles of *Mars*; A_W and A_E the observed right ascensions, reduced to the same instant; and D the approximate declination; then,

The horizontal parallax of *Mars*

$$= \frac{(A_E - A_W)\cos D}{\cos\psi_W \sin h_W - \cos\psi_E \sin h_E}$$

Again, if D_N and D_S denote the declinations observed at two stations differing very considerably in latitude (N denoting the northerly and S the southerly station, and the rest of the notation being suitably changed),

The horizontal parallax

$$= \frac{(D_S - D_N)\ \text{Sec. D}}{\sin\psi_N - \sin\psi_S + \tan D \cos\psi_S \cos h_S - \tan D \cos\psi_N \cos h_N}$$

This communication is accompanied by a list of stars selected by the author as favourably situated for observation at the ensuing opposition of *Mars*; the list includes many of those set down for a similar purpose in the *Nautical Almanac* for 1845.

VII. On the almost total disappearance of the earliest Trigonometrical Canon. By A. De Morgan, Esq.

I lately found in a second-hand book-shop a trigonometrical canon, by the celebrated Rheticus, which was totally new to me, and, as I afterwards found, would have been just as new to any of the historians of astronomy and mathematics. I am therefore induced to make a communication on the subject to the Astronomical Society, which, more than any other, has a right to take interest in the history of tables for the facilitation of the application of arithmetic to geometry.

The paper which I now read is a remote consequence of the proscription of Copernican opinions by the Inquisition. No follower of Copernicus was more zealous or more plain-spoken than Rheticus or George Joachim of Rhætia. The master, whether from conviction or policy, called his theory no more than an hypothesis for the explanation of the planetary motions: the pupil strongly insisted on the absolute physical truth of the motion of the earth. Both were gathered to their fathers long before the storm arose: but long before that time arrived, there was a distinction drawn between the treatment of the two. In the *Index Expurgatorius*, it is not Copernicus who is forbidden to be read generally; the prohibition only extends to the work *de Revolutionibus*, and is accompanied with a *nisi corrigatur*. But Rheticus is wholly forbidden to be read in any of his works: nay, in Sotomaier's folio edition of the *Index* (Madrid, 1667) even the *Opus Palatinum*, though all but the very table is the work of Valentine Otho, is an unlawful book, unless on the condition that the praises of Rheticus in the preface be erased or passed over. And Riccioli tells us that

the condemnation of the books of Rheticus took place in 1550, which is the year before the canon I intend to describe was published. The extreme severity with which his writings were treated will appear less remarkable if we remember that he held the chair of mathematics at Wittemberg, in which Luther had very lately taught theology, and burned the pope's bull, and in which Melancthon actually taught Greek. Riccioli very gravely informs us that Rheticus received a supernatural punishment for his presumption. While he was puzzling himself about the motion of Mars, he invoked his genius or guardian angel to help him out of the difficulty: the angel accordingly lifted him up by the hair of his head to the roof, and threw him down upon the pavement, saying with a bitter laugh, "That's the way Mars moves." Kepler, it seems, had heard this story with such evidence as induced him to suppose that Rheticus must have knocked his head against the wall in the agitation of his spirits. Riccioli does not feel himself at liberty to make any other mention of him, and adds *damnatus auctor* to his name in a list in which Copernicus himself figures without remark: and Clavius, in the diffuse trigonometry attached to the edition of Theodosius, does not even introduce his name. The Jesuit Blancanus also, in his list of mathematicians, excludes Rheticus altogether, though he was a contemporary of the *Opus Palatinum*, and though he admits Copernicus.

The works of Rheticus are accordingly very scarce; and it can be no matter of surprise that one of them, and perhaps more, should have entirely slipped out of notice. Even the second edition of Copernicus, Basle, 1566, which was edited by Rheticus, is much more scarce than the first.

The early history of trigonometrical tables, as given, may be thus summed up. Tables of sines, which were substituted for Ptolemy's chords by Albategnius, were published, to every minute, from Regiomontanus* (who died in 1476), and also by Apian, both in 1533. Regiomontanus used tangents, as the Arabs had done before him, and there is a table, to degrees only, in a work of his published by Reinhold in 1561. This same Erasmus Reinhold had printed tangents of every minute, in his *Liber Tabularum Directionum*, published in 1554, three years after the Prutenic tables, on which his fame principally rests. Regiomontanus had called

* Such is the statement: but on examining the work on triangles by Regiomontanus, in which these sines are said to occur, and which was published in 1533, I cannot find any table at all. But Apian published *two* tables of sines: one in the *Introductio Geographica*, 1533, and another in the *Instrumentum primi mobilis*, 1534. There is a table of sines in the *Tabulæ Directionum*, 1552, and another in the *second* edition of the work *De Triangulis*, Basle, 1561, folio. And in the catalogue of De Thou's library, and in Niceron's list, Murhard's catalogue, and Kästner's history, is mentioned the following work, "Joh. Regiomontani compositio tabularum Sinuum, cum tabulis duplicibus Sinuum ejusdem," folio, Nuremberg, 1541. All the writers who assert that sines of Regiomontanus were published before those of Apian, go on the assumption that what is in the second edition of the work on triangles is also in the first, which assumption is not true. Lalande gives a wrong date, place, and title to this first edition.

the table of tangents *canon fœcundus*, which name Reinhold adopted. As to secants, the table of which was afterwards called *canon fœcundissimus*, a name which I think first occurs in Vieta, the first table mentioned is that of Maurolycus in the spherical treatise which accompanies his edition of Theodosius and Menelaus, published in 1558; this table goes only to degrees, and is called *tabula benefica*. The *Canon Mathematicus* of Vieta, published in 1579, is said to have been the first work in which sines, tangents, and secants, were joined together; that is, the first complete canon printed. But all admit that Rheticus, who died in 1576, had very nearly completed the enormous table which Valentine Otho published in 1596, under the name of *Opus Palatinum*. Still, the rigid rule is, that first publication gives a right which nothing but unquestionable proof of fraud can impugn; and accordingly Vieta has been justly considered, up to this time, as the first author of a complete canon. I intend to shew, however, that under the same rule, Rheticus is not only the first who published either tangents or secants,* but the first who joined the three into a complete canon, and also the first who adopted the now universal semi-quadrantal form. Before, however, I come to the description of the table which establishes these things, I shall shew that it once had a recognised existence. It is not enough, if better may be, to produce a printed book as the sole evidence of the fact of publication. There may have been a suppressed edition, or one accidentally destroyed by fire, and of which only a few copies escaped: the forgery of a work is neither impossible nor unexampled, and more than one big catalogue consists entirely of pseudonymous works.

Lansberg attributes the first publication of tangents, and Bossut the first publication of secants, to Rheticus: on what authorities I do not know. Moestlinus, in a letter written to Kepler in 1594, and published in the folio correspondence of the latter, expresses a great desire to see a certain little book of Rheticus, the title of which he does not mention. In a subsequent letter, of 1595, he reports that he has seen this book, that it treats only of plane triangles, and that it strives at (*nititur*) the canon of triangles which Rheticus afterwards (*olim*) published. What can this *canon of triangles* be? At the date of this last letter, the *Opus Palatinum* was not published. It can refer to nothing but the work which I now bring forward, and which, it thus appears, was known. The next witness

* Montucla gives the secants to Rheticus, and Delambre (*Astron. Mod.* ii. 34) seems to assent, because Maurolycus only published them to every degree. This is hardly fair: a person who points out the uses of a given function, and tabulates it to a certain extent, is the inventor, and must not lose his right because another gives more and better tabulation. But we now see that Rheticus has a claim absolutely prior to that of Maurolycus. Again, Delambre says that a certain Finckius gave secants to minutes, in 1583, referring to Rheticus: he seems to imply that Finckius had access to the materials of the *Opus Palatinum*, not then published. Perhaps it may now be held that the ten-minute canon described in the present article was the original, the intermediate minutes being supplied by interpolation.

I shall cite is Thomas Digges, who is likely to have taken a particular interest in the writings of his fellow Copernican. In the *Alæ seu Scalæ Mathematicæ*, published in 1573, during the life of Rheticus, Digges says that those who dislike labour should consult the tables of the proportions of right-angled triangles by Rheticus, of which tables he further states that they go to every ten minutes.

Valentine Otho, in the preface to the *Opus Palatinum*, says that " when Rheticus gave to the public a specimen of his method of enriching the canon and doctrine of triangles, he excited a wonderful degree of hope and expectation about it in the minds of the learned, especially when in the dialogue which he prefixed to his ten-minute canon, he brought forward extraordinary and almost incredible things concerning its use." I produce to the Society the ten-minute canon with the dialogue, which however is not prefixed to the table, but follows it. Otho afterwards mentions this dialogue again, and says that on reading it he was so struck with the pretensions of its author that he sought out Rheticus in Hungary, and commenced the acquaintance which led to his becoming the editor of his friend's posthumous work. He further adds, that at their first meeting, when he had just stated that he was come to acquire knowledge on the properties of triangles, Rheticus interrupted him with, " You are just as old as I was when I went on the same errand to Copernicus." Thus it appears that this canon was the indirect cause of the publication of the *Opus Palatinum*.

But nevertheless, this work is not mentioned in the catalogues of Lipenius, Dechâles, or Murhard; it is not alluded to either by Riccioli, Clavius, Gassendi, Weidler, Heilbronner, Delambre, Montucla, Hutton, or Kästner. Delambre distinctly says he never heard of any canon containing sines, tangents, and secants, previous to that of Vieta: though elsewhere he describes the preceding passage from Otho by saying that Rheticus had published a programme of the *Opus Palatinum*, and even an extract for every ten minutes. But he then quotes the account of Lalande, to which I shall immediately come, and drops the matter, as if declining to decide the point. Kästner quotes the passage from Otho which I have translated above, and makes a separate head of it, in his abstract of the preface of the *Opus Palatinum;* he does not give his usual short comment, and evidently leaves it to the reader without knowing what to make of it.

That Weidler should have been utterly ignorant of this work is rather a striking proof of the complete oblivion into which it had fallen. For Rheticus (with Reinhold) professed mathematics in the University of Wittemberg, after having taken degrees there; and Weidler was not only of this university, but wrote his history of astronomy, and printed it, at Wittemberg. Consequently he had access to the *matricula*, or register-book, and to all other records; so that he is able to give several minute particulars of the literary life of Rheticus, which another writer could hardly have obtained. But not even at Wittemberg did any tradition exist of the work on which this paper is written. It was printed, certainly, at Leipsic,

and on the residence of Rheticus in this latter university, Weidler can say nothing more than that Rheticus is reported to have taught there. It is worth noting that, next to Wittemberg, Leipsic was the university most obnoxious to the adherents of the old church.

Lalande actually possessed a copy of a reprint of this same canon, published (he says) in 1580, and has given the title-page with perfect correctness, in his short description of the *Opus Palatinum*, contained in the *Bibliographie Astronomique*. But, with a negligence which is unusual in his bibliographical accounts, he represents it as a canon for the first forty-five minutes only, to every ten seconds; a kind of extract from the forthcoming *Opus Palatinum*. Had he looked more closely, he would have seen that his *minutes are degrees, and his seconds minutes;* and he would have seen the remaining 45 degrees rising in the reverse direction on the opposite side of the page. Perhaps Lalande could not imagine the possibility of the calculator of the immense *Opus Palatinum* publishing a table to every ten minutes only.

Murhard has this reprint, Basle, 1580, in his list, but not the original work, and it is also in the catalogue of De Thou's library. The title is to be found in Teissier's *Eloges des Hommes Savans*, from whence it is copied into a work in which we should hardly have looked for mathematical treatises which are unknown to mathematicians, Gorton's *Biographical Dictionary*. The library of the British Museum, which is unusually rich in the mathematical works of the sixteenth century, has both the original work and the reprint. But I do not find any date to the reprint, nor do I know from whence Murhard and Lalande got theirs. The copy of the original edition now in my possession probably escaped the Inquisition from the accident of its being bound up with the *editio princeps* of the Greek text of the optical writings of Euclid, published six years after it.

We thus see that the existence of a work may be forgotten, and the fair claims of its author reduced in amount, by the neglect of a biographer in stating precisely its title, date, place, and form; and also that it may be possible, partially at least, to repair the neglect, by collection of scattered notices. We also see that the publication of proper catalogues of our libraries would tend to promote historical knowledge. And while on this subject, I trust it is not out of place to make the following remark. I very much fear that the publication of a good alphabetical catalogue of the splendid library in the British Museum is retarded by the demand which has often been made for a *classed catalogue*, or one arranged in order of subjects. From much, almost daily, use of catalogues for many years, I am perfectly satisfied that a classed catalogue is more difficult to use than to make. It is one man's theory of the subdivision of knowledge, and the chances are against its suiting any other man. Even if all doubtful works were entered under several different heads, the frontier of the dubious region would itself be a mere matter of doubt. I never turn from a classed catalogue to an alphabetical one without a feeling of relief and security. With the

latter I can always, by taking proper pains, make a library yield its utmost: with the former, I can never be satisfied that I have taken proper pains, until I have made it, in fact, as many different catalogues as there are different headings, with separate trouble for each. Those to whom bibliographical research is familiar know that they have much more frequently to hunt an author than a subject: they know also, that in searching for a subject, it is never safe to take another person's view, however good, of the limits of that subject, with reference to their own particular purposes.

I now proceed to the description of the canon.

The title is, "Canon doctrinæ triangulorum. Nunc primum a Georgio Joachimo Rhetico, in lucem editus, cum privilegio imperiali, Ne quis hæc intra decenniom, quacunq; forma ac compositione, edere, neue sibi uendicare aut operibus suis inserere ausit. Lipsiæ ex officina Wolphgangi Gunteri. Anno M.D.LI." quarto. The title is followed by one page of verses, fourteen of tables, and six of dialogue. In the title-page is an obelisk, with a man drawing a diagram on the base. The degrees and the sines, &c., are in black ink, the minutes and the differences are in red ink. The tables extend across the open book, the $\frac{\text{sine}}{\text{cosine}}$, $\frac{\text{cosine}}{\text{sine}}$, and $\frac{\text{secant}}{\text{cosecant}}$, being on the left; the $\frac{\text{tangent}}{\text{cotangent}}$, $\frac{\text{cosecant}}{\text{secant}}$, and $\frac{\text{cotangent}}{\text{tangent}}$, on the right. Thus at 30° 0′, we read along the double page, as follows, all in one line, differences coming before the semicolon which is here substituted for the double line in the work.

30 0 5000000 25171; 8660254 14581;
11547005 19474; 5773503 38851;
20000000 100180; 17320508 115774; 0 60.

The words sine, tangent, &c., are not used. The running title is, "Canon doctrinæ triangulorum in quo triquetri cum angulo recto in planitie partium 10000000 ponitur." This running title is not a complete sentence, the nominative of the last verb being wanted: it is completed in three different ways in the three great compartments of the table; the three nominatives are "Subtendens angulum rectum,"—"Majus latus includentium angulum rectum,"—"Minus latus includentium angulum rectum." Under the first heading, come four columns, the *perpendiculum* and its *differentia*, and the *basis* and its *differentia*. Under the second, come the *hypotenusa* and *perpendiculum*; under the third the *hypotenusa* and *basis*: all with differences. The words *perpendiculum* and *basis* are of course interchanged in the running titles at the bottom of the page, where also no other part of the running title is repeated. This system of headings is faithfully preserved in the *Opus Palatinum*.

Modern teachers of trigonometry have pretty generally abandoned the system of independent lines, which used to be called sines, tangents, &c.; and have substituted, for the meaning of these words, the ratios of the sides of right-angled triangles. It appears that they have antiquity in their favour; indeed so completely has the idea of representing the ratios of the sides of triangles taken possession of the

mind of Rheticus, that he abandons the use of the word *sine*. He dwells on the importance of the right-angled triangle, without any reference to the circle: his maxim, expressed in the dialogue, is *Triquetrum in planicie cum angulo recto, est magister Matheseos.* It would also seem as if his choice of the semiquadrantal arrangement with double descriptions was dictated merely by the convenience of heading one division with *majus latus*, and the other with *minus latus*. This is worth noting: most persons suppose that this disposition must have arisen from the circumstance of the sines and cosines of the latter half of the quadrant being only repetitions of the cosines and sines of the first half, and so on. But the very reverse is the fact; the names cosine, cotangent, and cosecant, are the consequence, not the cause, of this duplicate system of arguments. Rheticus made his arrangement with a view to separate from each other all the cases in which the greater side and the less side were data. This involved the bringing into the same line the tangent of each angle with the tangent of its complement, and the same for the secant. Completeness then required that the same should be done with the sine. The introduction of the terms, sine of the complement, complemental sine, and cosine, &c., followed after an interval of more than half a century, and was a consequence of the semiquadrantal arrangement.

The dialogue at the end is between Philomathes, a supposed friend of Rheticus, and Hospes, his pupil. The pupil asks what the intention of the book is, and is answered at length. He suggests that, perhaps, the intention may be to complete the system of Copernicus, by publishing tables from it resembling those then in use. But he is answered that Rheticus would rather that Copernicus himself had not done so much in this line, as he thereby diminished the geometrical practice of the learner. The modern astronomer, if such a one there be, whose luxurious means render him discontented whenever he has to go to the common trigonometrical tables, even of logarithms, should think of Rheticus, so well content with his intervals of ten minutes, and their differences, that he asked for nothing more, and regretted that any further help should be interposed between the observer and his wholesome exercise. I think I might have hoped for this sanction to an opinion which has been for a long time my own, namely, that the only way of learning to use a table thoroughly well, is to learn to do without it.

There is much examination yet wanted into the history of the sixteenth century. The era of logarithms, of literal algebra, and of sound mechanics, has naturally diverted attention from the day of smaller things. One question has never been properly considered: what were the immediate producing causes of that burst of successful energy which marks the first half of the seventeenth century?

The biography of Rheticus may be collected from the preface to the *Opus Palatinum*, Teissier's *Eloges des Savans*, with the references therein given, &c. He led a wandering life for a calculator of such a mass of tables, being successively with Copernicus, at Wittemberg, at Leipsic, and in Hungary. And his friend and

correspondent, Peter Ramus, informs us (*Schol. Mathem.*) that he taught at Cracow, and would have taught at Paris, but was prevented by being obliged to learn and practise medicine in the stead of a certain patron, *Mæcenatis cujusdam loco.* What this means I do not know: perhaps *loco* is a misprint for *domo.* Though Otho does not mention that Rheticus practised medicine, he to a certain extent confirms Ramus, by stating that his friend died at Cassau, in Hungary, on his way home, after being called out by a certain baron. I mention these things, because it is never stated that Rheticus was a physician. He died in 1576, in the sixty-first year of his age.

With regard to the choice of intervals of ten minutes, it may have been dictated by the existing state of astronomy; but it is more than likely that Copernicus was the suggester of the arrangement. Rheticus has preserved it as a saying of Copernicus, that if he could only succeed in giving planetary tables which should be true within ten minutes, he should feel as much gratified as Pythagoras, when he discovered the great property of the hypothenuse of a right-angled triangle.

In this communication I have confined myself to points which are either new, or very little known. I will add one more circumstance of the latter kind.

I have noted that Rheticus stands in the *Index* as *damnatus auctor*, while Copernicus is *damnati libri auctor:* a material difference. But perhaps it may suggest itself to some that Copernicus is only the writer of one work, which, being condemned, makes him a condemned author: and that it would not be thought necessary to condemn, in general terms, a writer all whose works can be prohibited under one title. But, not to dwell here upon such a supposition really implying an ignorance of the usage, it is not true that Copernicus wrote and published only one work. Though it be but little known, and not mentioned by any of the French school of historians, it is certain that Rheticus himself published (Wittemberg, 1542) in 4to. the "De lateribus et angulis triangulorum tum planorum rectilineorum, tum sphæricorum libellus," containing a table of sines to every minute, and to a radius of ten millions, or, as we should now say, to seven decimals.

This work is mentioned by Weidler, who, though of Wittemberg, had no knowledge of it till he published the supplements of his history: it is catalogued by Murhard, and described by Kästner. The great work *de Revolutionibus*, published the next year, contains a probably abridged treatise on triangles, and a certainly abridged table of sines.

In speaking of the great work of Copernicus, it should be remembered that Rheticus procured its publication, or extracted the author's consent to its appearance, as much as Halley did that of Newton in the case of the Principia; and nothing but circumstances which made it more convenient to print it at Nuremberg than at Wittemberg, prevented the name of Rheticus from appearing in the title-page as editor.

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